CZECH METROLOGY INSTITUTE

Units of electrical impedance, their traceability to quantum Hall effect, digital bridges

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Contents

- Motivation
- Some history of units
- DC resistance at CMI
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- How to link impedance units to quantum Hall effect
- Primary impedance bridges for scaling units at CMI
- Realization of AC quantum Hall effect at CMI
- Graphene based devices and cryocooling
- Summary

Motivation

- ► **Linkage of electrical impedance units** for *R*, *C*, *L* directly **to reference resistance standards**
  - ✓ One system
  - ✓ Few calibration steps
  - ✓ Few reference standards

AC QHR

AC Resistor

+

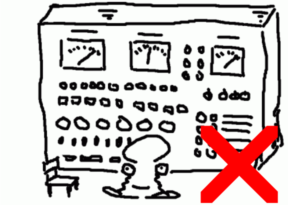
Digital impedance bridge

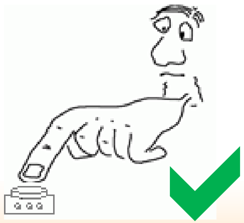
+

Reference clock

→

R, C, L

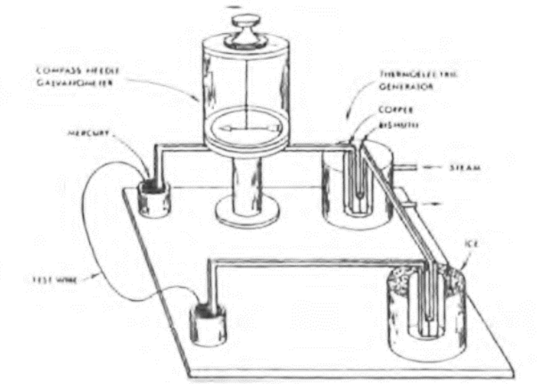




History

► DC resistance

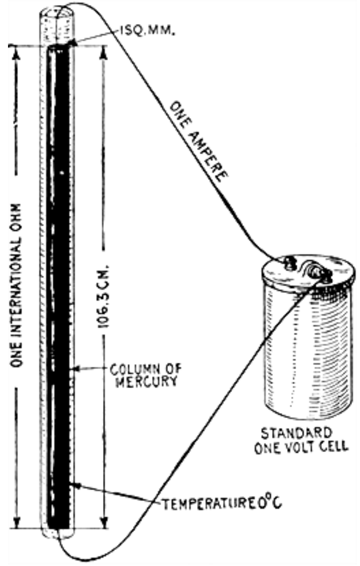
Investigation of I=U/R relationship ~ 1826



1-1 Ohm's Circuit for Measuring Resistance 1826

[G. S. Ohm, 1826, The Galvanic Circuit Mathematically Worked Out]

First International Ohm - 1893

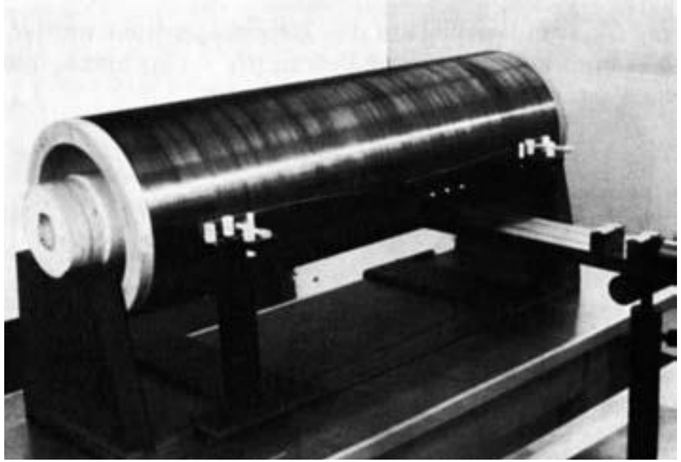


[N. Hawkins: Hawkins Electrical Guide vol.8, Theo. Audel & Co., 1914, www.gutenberg.org]

History

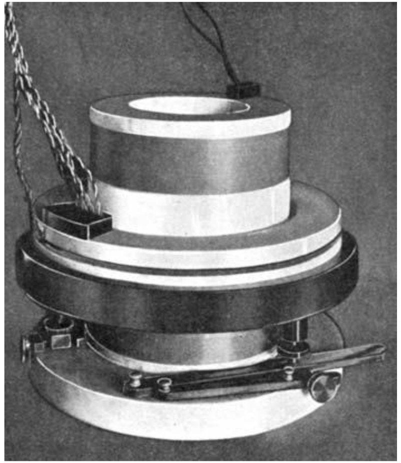
▶ Realization of inductance

PTB's design 100 mH selfinductor



[Linckh and Brasack 1968, IOPP]

NPL's mutual inductor



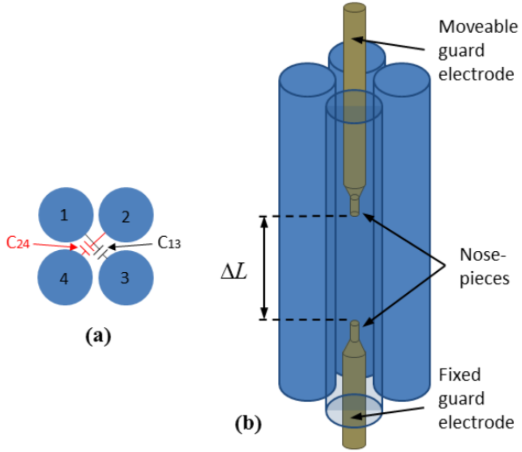
[Campbell and Childs,1935, IOPP]

- **Derivation of ohm from henry till 1970s**

History

▶ Realization of capacitance

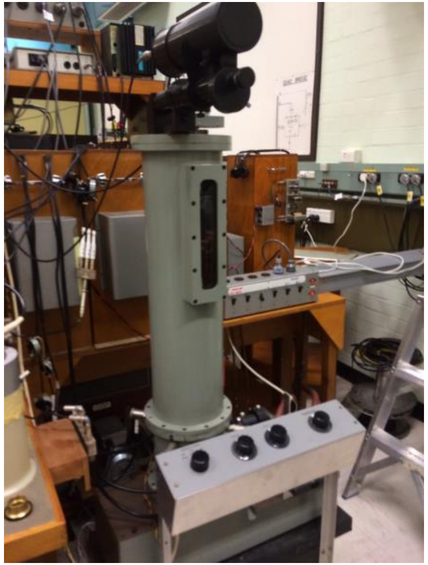
- Thompson – Lampard theorem ~ 1956



$$\Delta C = \varepsilon_0 \; (\mathrm{Ln}2/\pi) \; \Delta L$$

[Gournay *at al.* 2015, ICM]

Original capacitor at NMIA, Australia

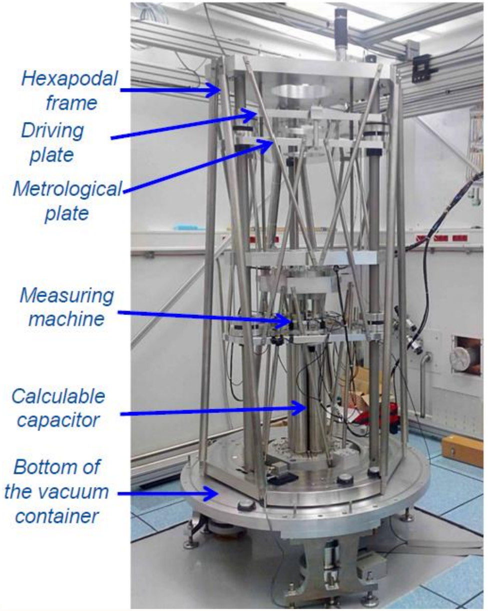


- **Derivation of ohm from farad**

History/Today

► Realization of capacitance

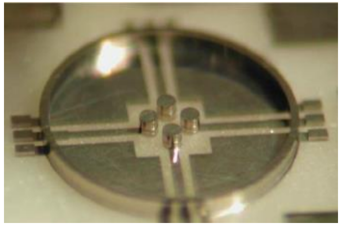
New design LNE (France)



- ✓ **2.4 m** high and **1.2 m** width
- ✓ **5** electrodes
- ✓ Capacitance variation of **1 pF**
  ΔL = 370 mm
- ✓ Motor with encoder (resolution **55 nm**) and displacement measured with a Michelson interferometer (resolution **0.3 nm**)
- ✓ 532 nm laser source tuned on an I_2 line

[Piquemal *at al.* 2015, CODATA Workshop]

3D MEMS Calculable Capacitor

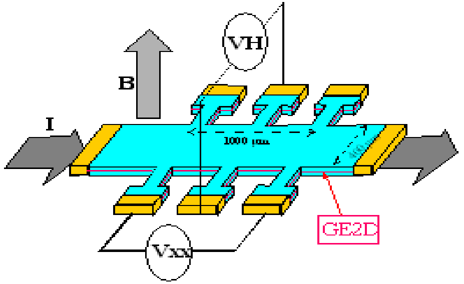


[Awan *at al.* 2005]

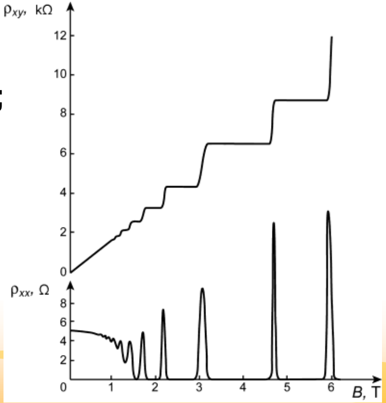
History/Today

► DC resistance

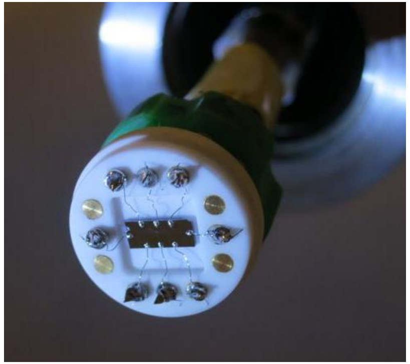
K. von Klitzing – Nobel prize 1985



R_H **(i) = h/*i*e^2 ; *i* = 1,2,3,…;**



PTB's GaAs/AlGaAs based device



CMI's fondant based device

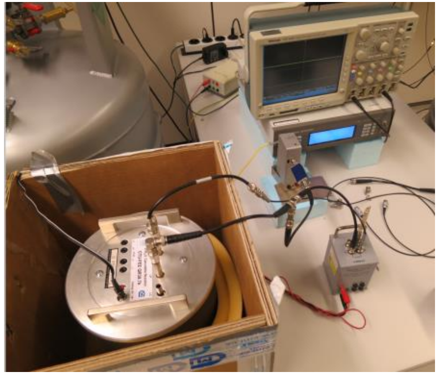
Today:

- **Quantum Hall arrays (1 kOhm, 1 MOhm)**
- **Graphene based devices**
- **Quantum Hall effect in topological insulators**

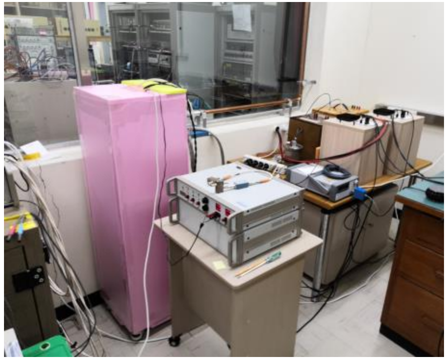
History/Today

► Resistance and capacitance standards with calculable frequency dependence

Calculable resistance standards

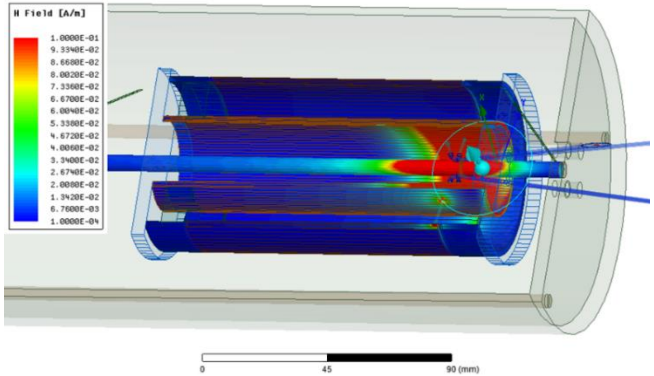


CTU/CMI 40 kOhm at PTB JVS lab



CTU/CMI 12.9 kOhm at KRISS lab

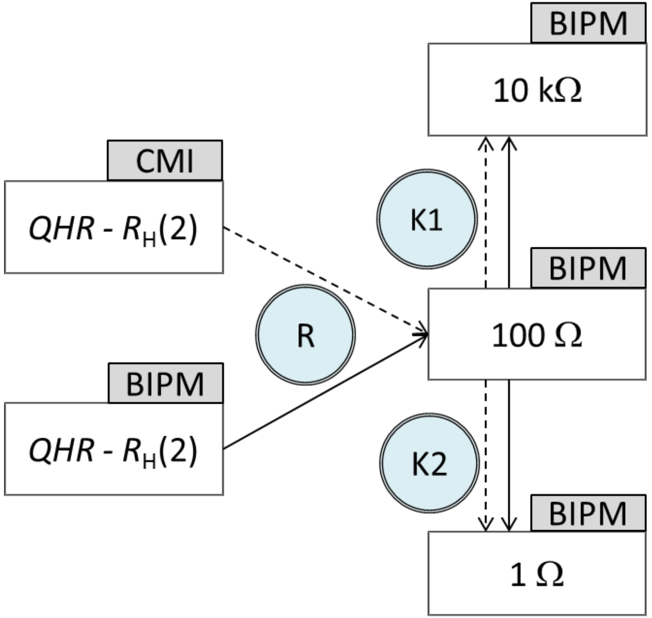
Calculable capacitance standards



[J. Boháček and B. M. Wood, "Octofilar resistors with calculable frequency dependence," *Metrologia* , vol. 38, no. 3, 2001]
[J. Kucera, R. Sedlacek and J. Bohacek, "Improved calculable 4TP coaxial capacitance standards," CPEM Digest 2014]
[L. Vojáčková, J. Kučera, J. Hromádka and J. Boháček, "Calculation of high frequency 4-TP impedance standards," ," CPEM Digest 2016]
[Kim D B, Kassim D M, Kim W, Callegaro L, D'Elia V, Trinchera B, Kucera J and Sedlacek R, "Traceability Chain at KRISS from DC Quantum Hall Resistance to Farad Using Coaxial Bridges," *IEEE Trans. Instrum. Meas*, 2019]

Today

▶ CMI: On-site comparison with BIPM (2017)



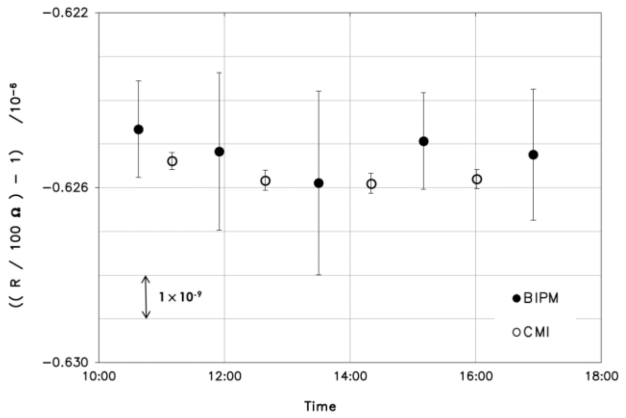
Comparison scheme

BIPM's standards

Today

▶ DC resistance at CMI

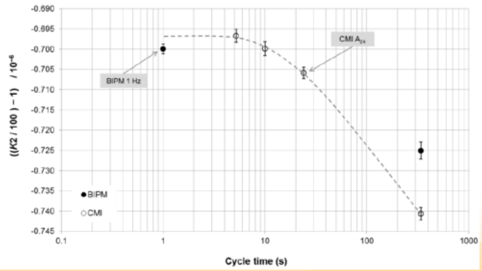
BIPM.EM-K12, QHR→100 Ω



Comparison results

	Degree of equivalence D /10^{-9}	Expanded uncertainty U /10^{-9}
$R_{100\Omega}$ in terms of $R_{\mathrm{H}}(2)$	−0.6	5.0
$K1 = R_{10k\Omega}/R_{100\Omega}$	+1.1	4.4
$K2 = R_{100\Omega}/R_{1\Omega}$	+3.3	6.4

Raised 1 Ω problem: what is “DC”?



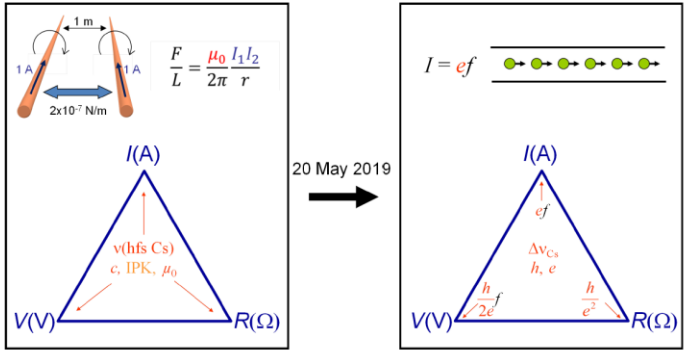
Redefinition of SI units

- **Problem:** … because the electric units refer to the force and thus to the kilogram through the ampere definition, a drift of the kilogram induces a similar drift in the electrical units ….
  … Also in temperature measurement, the previous definition of the base unit kelvin via the water triple point cell (type 2 according to the classification above) reaches its limits ….

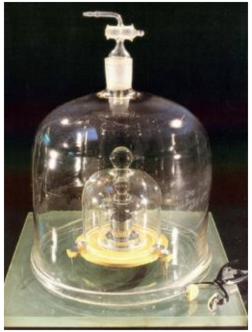
[Jeckelmann B., Progress in Physics (65) 2018]

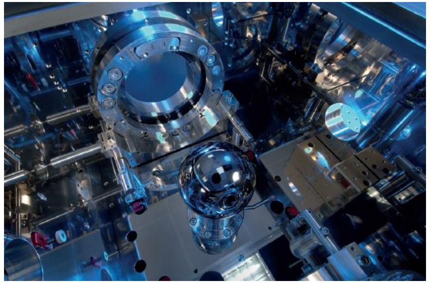
Precondition for revision:

1. XRCD experiment: refers kilogram either to an atomic mass or to the Planck constant
2. Kibble balance experiment: compares mechanical and electrical power. If the electrical power is measured with quantum standards (R_K, K_J), the mass can be related to Planck's constant

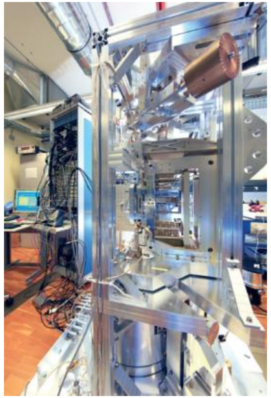


Ampere definition





Silicone sphere
in PTB's interferometer

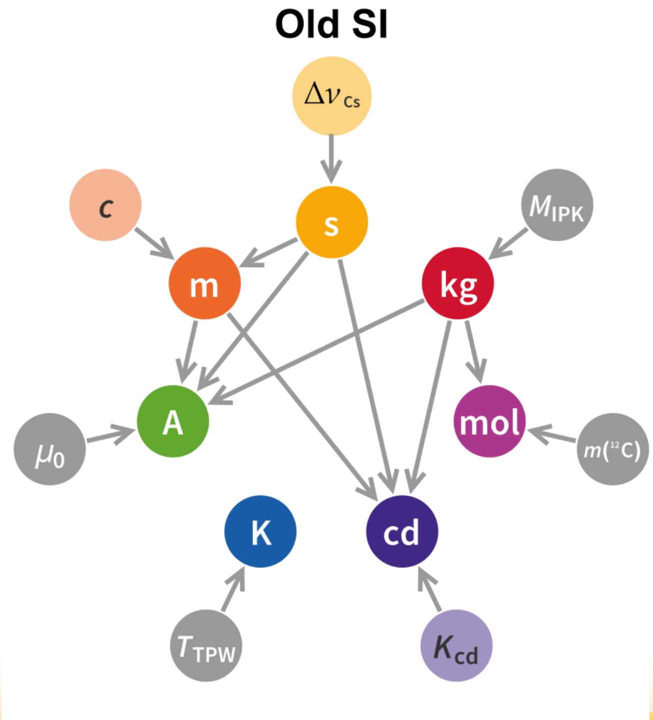


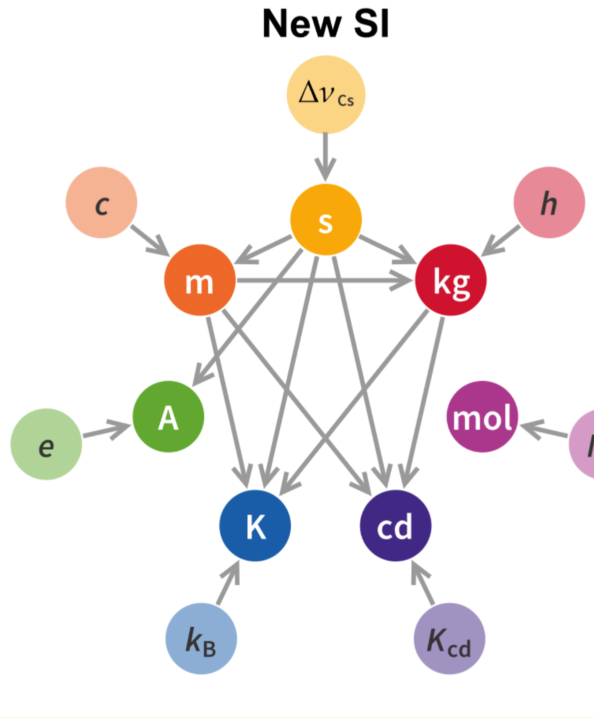
METAS' Kibble balance

[Poirier W, Djordjevic S and Schopfer F 2019, Comptes Rendus Physique 20 92–128]

Redefinition of SI units

- **20 May 2020: setting exact numerical values for the Planck constant (*h*), the elementary electric charge (*e*), the Boltzmann constant (k_B), and the Avogadro constant (N_A),**



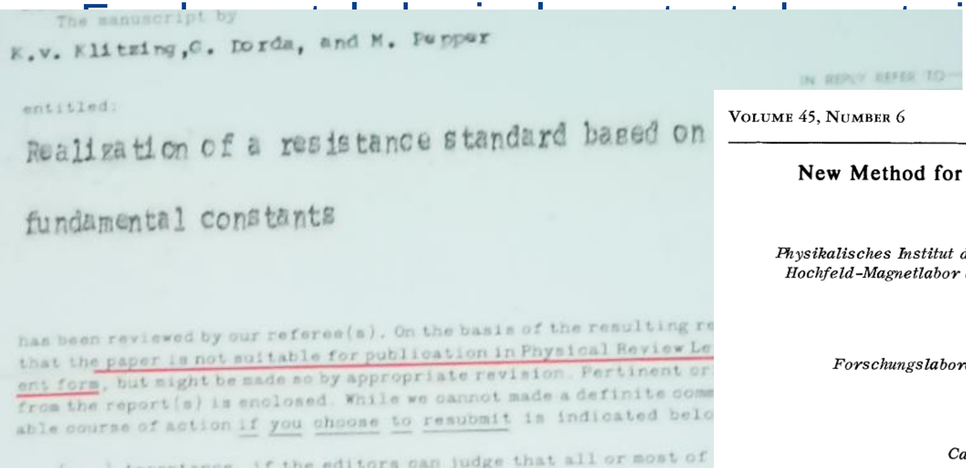


Redefinition of SI units

► **Fine structure constant (Sommerfeld's constant)**

zing the strength of the

entary charged particles.



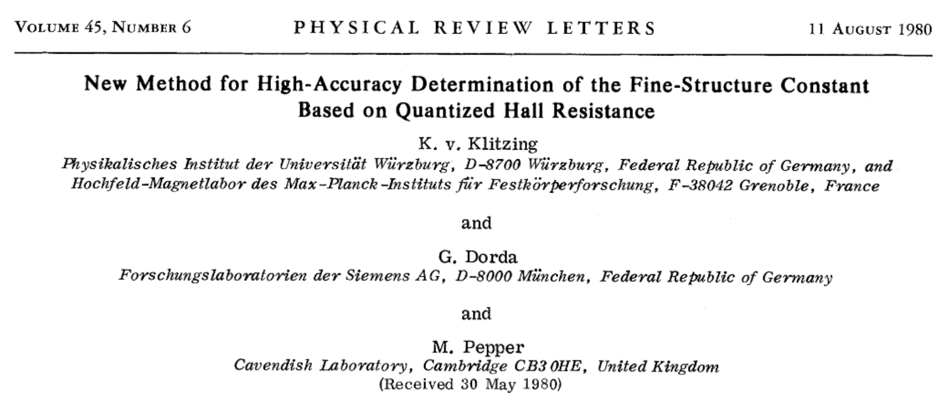
The manuscript by

K.v. Klitzing, G. Dorda, and M. Pepper

entitled:

Realization of a resistance standard based on fundamental constants

has been reviewed by our referee(s). On the basis of the resulting re
that the paper is not suitable for publication in Physical Review Le
ent form, but might be made so by appropriate revision. Pertinent or
from the report(s) is enclosed. While we cannot made a definite com
able course of action if you choose to resubmit is indicated belo



VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing

Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

and

G. Dorda

Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper

Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom

(Received 30 May 1980)

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar} = \frac{k_e e^2}{\hbar c} = \frac{c\mu_0}{2R_K} = \frac{e^2}{4\pi}\frac{Z_0}{\hbar}$$

$$\alpha^{-1} = \frac{2R_K}{\mu_0 c}$$

Redefinition of SI units

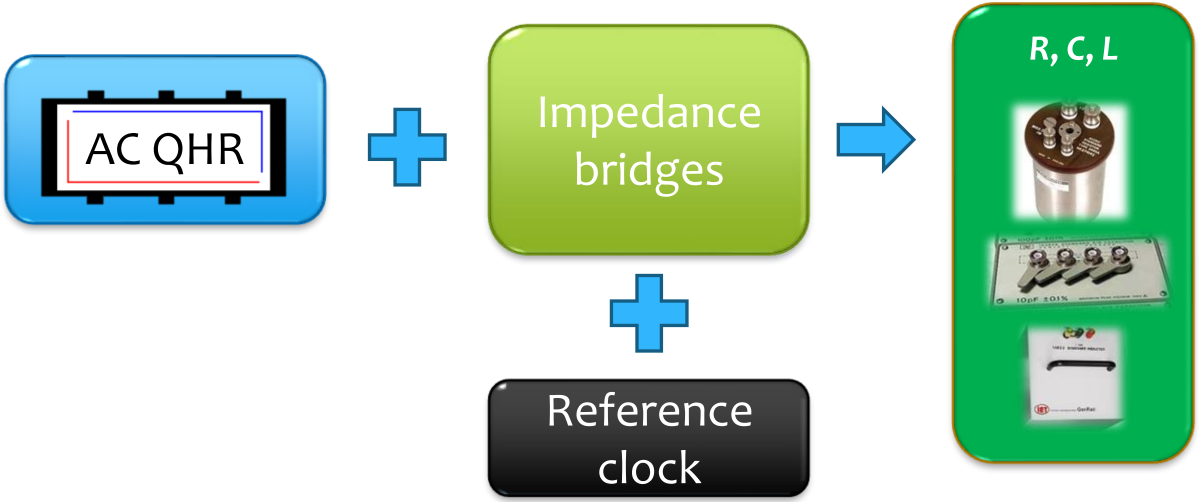
► **Change of electrical quantities since 1990**

Quantity	Formula for SI Unit	Relative Change
voltage	$V = V_{90}\,[1 - (100 \times 10^{-9})]$	-100 ppb
resistance	$\Omega = \Omega_{90}\,[1 - (17 \times 10^{-9})]$	-17 ppb
current	$A = A_{90}\,[1 - (83 \times 10^{-9})]$	-83 ppb
charge	$C = C_{90}\,[1 - (83 \times 10^{-9})]$	-83 ppb
power	$W = W_{90}\,[1 - (183 \times 10^{-9})]$	-183 ppb
capacitance	$F = F_{90}\,[1 + (17 \times 10^{-9})]$	17 ppb
inductance	$H = H_{90}\,[1 - (17 \times 10^{-9})]$	-17 ppb

[N. Zimmermann *et al.* 2015 NCSLI Measure J. Meas. Sci.,10(2)]

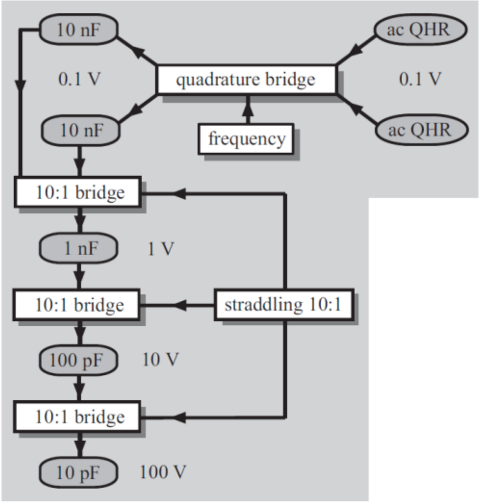
Linking impedance units to QHE

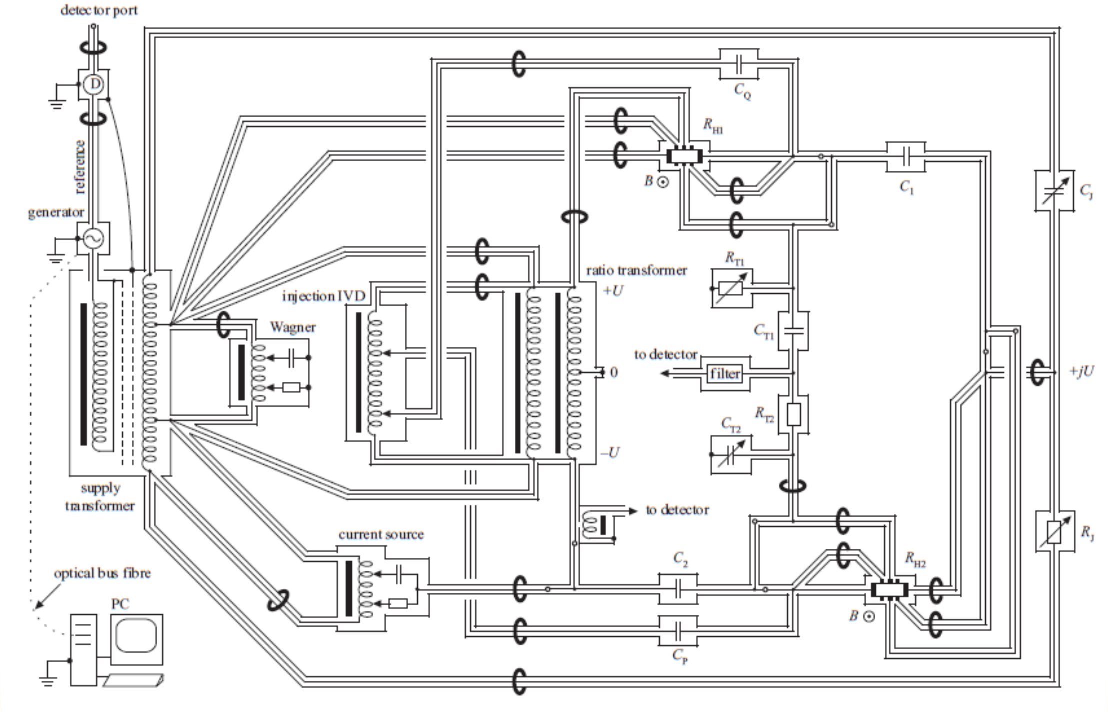
► **Linkage of electrical impedance units** for *R*, *C*, *L* directly **to reference resistance standards**



Linking R-C

► PTB’s quadrature bridge for 2 x C and 2 x QHR

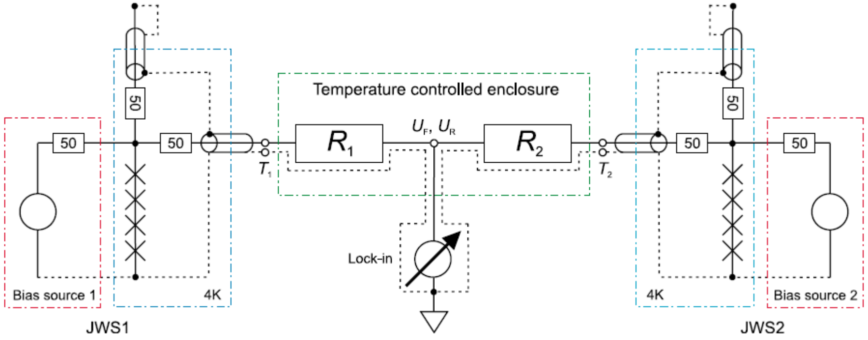


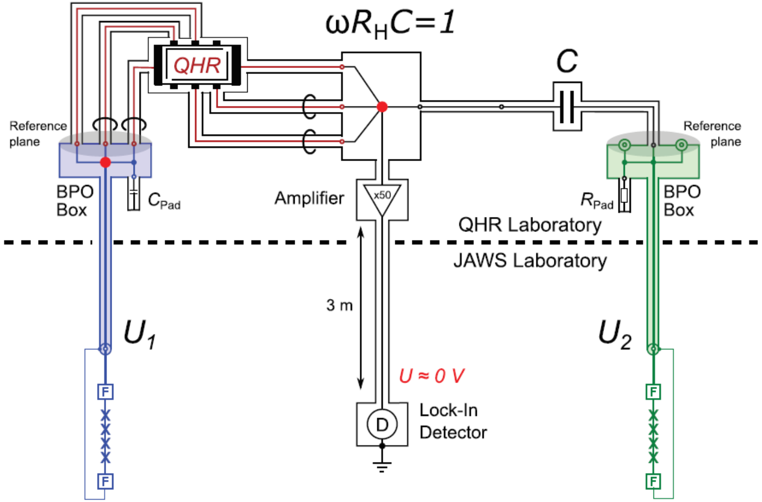


[S. Awan, B. Kibble, and J. Schurr, London, UK, 2011]

Linking R-C

► **Josephson based impedance bridges**





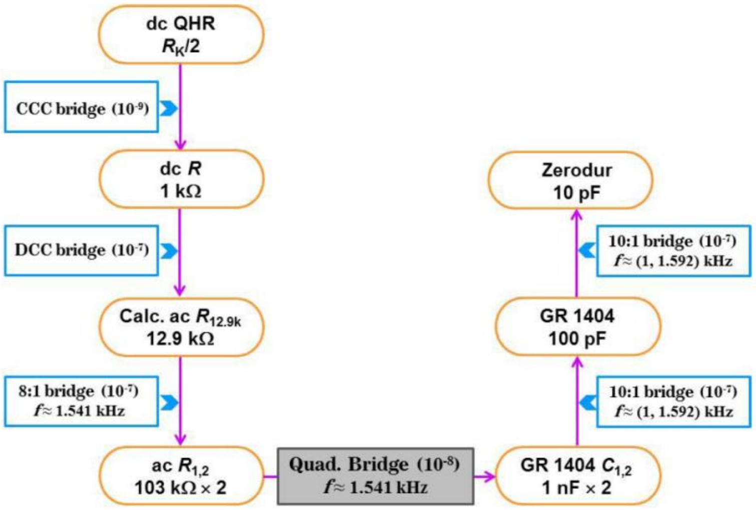
[J. Lee, J. Schurr, J. Nissilä, L. Palafox, and R. Behr, 2010 *Metrologia* 47(4)]

[S Bauer *et al* 2017 *Metrologia* 54 152]

Linking R-C

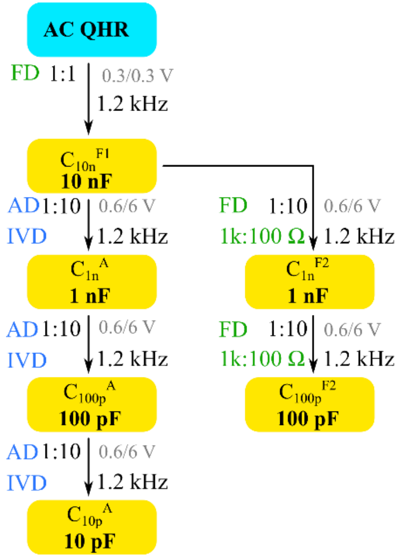
► **"Standard" chains of R-C traceability**

"Classical" chain
with 4-5 bridges and 2 calculable R



[Kim D B *at al.*," *IEEE Trans. Instrum. Meas*, 2019]

CMI's approach
with 1-2 bridges and 2 calculable R

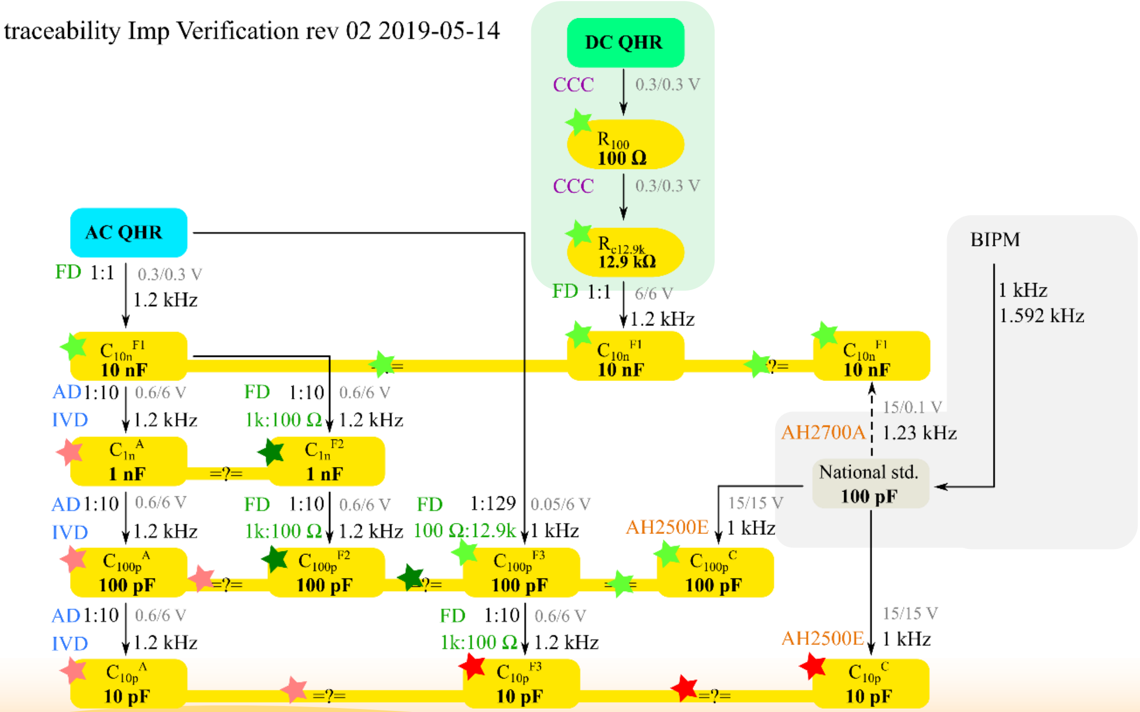


Linking R-L: with FD bridge too

[J. Kučera, P. Svoboda, J. Kováč, K. Pierz, *in preparation*]

Linking R-C at CMI

► Verification of traceability chain



Impedance bridges for scaling of units

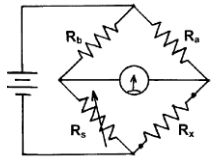
► **Calibration** of electrical impedance standards (R,C, L) on the highest metrological level in range of x Ω … x MΩ in audio frequency range is performed by means of **coaxial impedance bridges.**

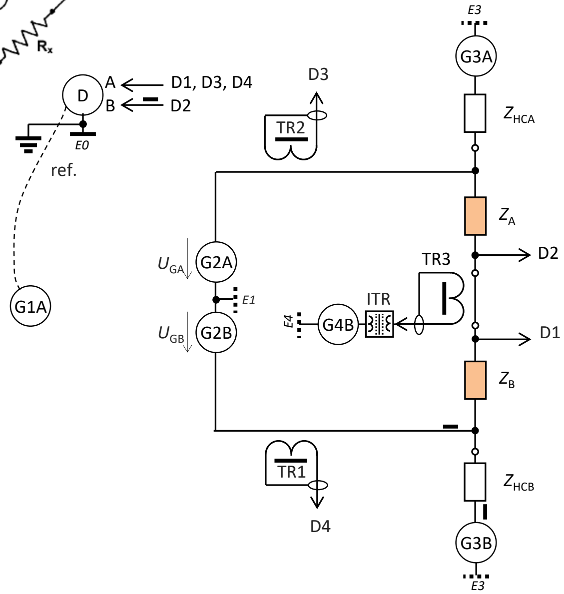
1) Manually operated bridges:
- rel. accuracy in the range 10^{-7} up to 10^{-9}
- manual operation
- coverage of only predefined ratios

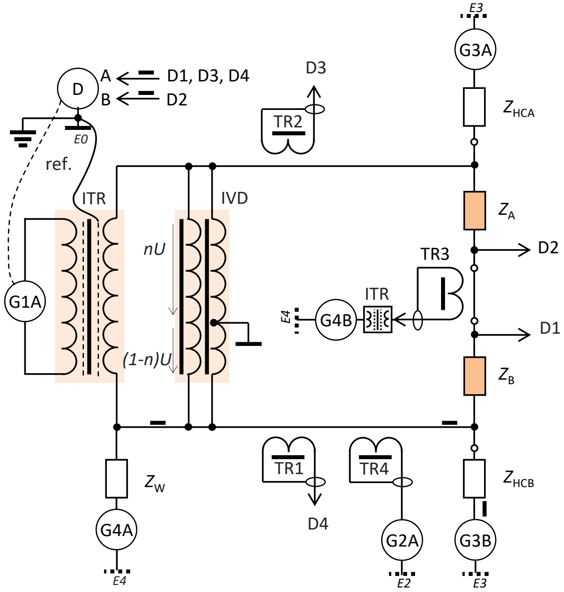
2) Digitally assisted bridges (DA):
- rel. accuracy in the range 10^{-6} up to 10^{-8}
- partial/full automation
- coverage of only predefined ratios

3) Fully digital bridges (FD):
- rel. accuracy in the range 10^{-5} up to 10^{-7}
- full automation
- coverage of the whole complex impedance plane

CMI's Fully digital / Digitally assisted bridge







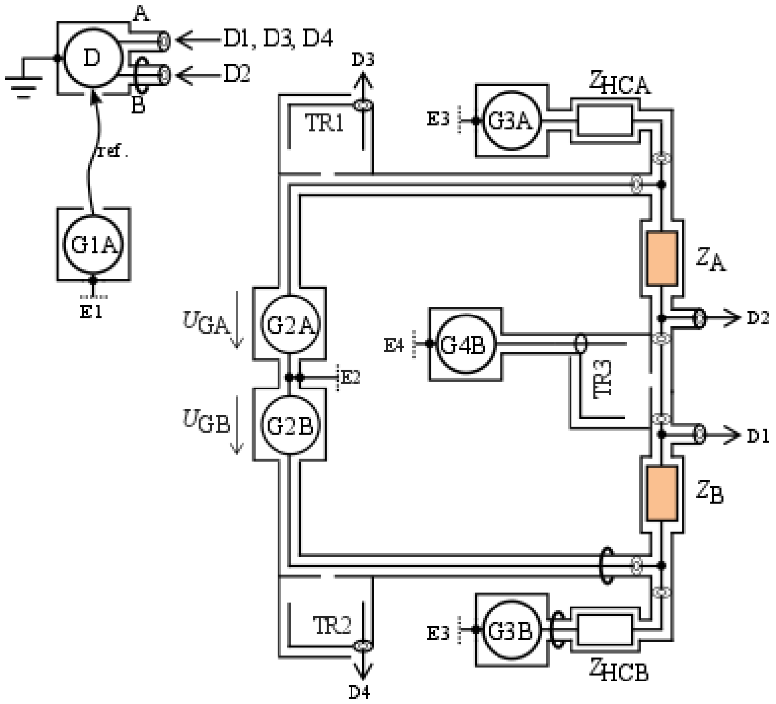
$$Z_B / Z_A \approx U_{GB} / U_{GA}$$

$$Z_B / Z_A \approx [1 - n + U_{inj} / U] / n$$

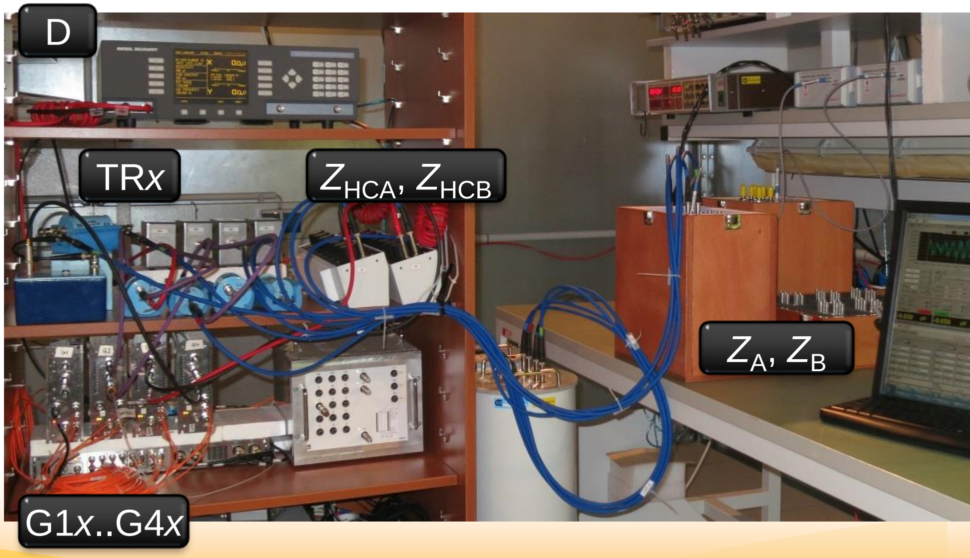
FD bridge

DA bridge

4-TP Fully digital bridge



Four terminal pair conditions:
U_{D1}=0
U_{D3}=0
U_{D4}=0
(U_{D2}-U_{D1})=0



$$Z_B / Z_A \approx U_{GB} / U_{GA}$$

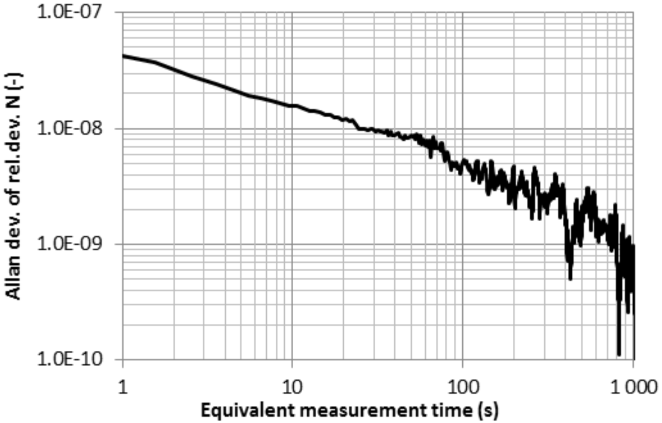
DigiBridge realization

- **Isolated generators** SWG
  - No jumps during setting U, ω, 0
  - User defined voltage sweeps
  - Full balancing to zero

  [J. Kučera and J. Kováč, *IEEE Trans. Instr. Meas*, vol. 67, no. 99, 2018]

- **Multiplexers**
  - crosstalk < -185 dB

  [J. Kováč, J. Kučera, XXI IMEKO, 2015]



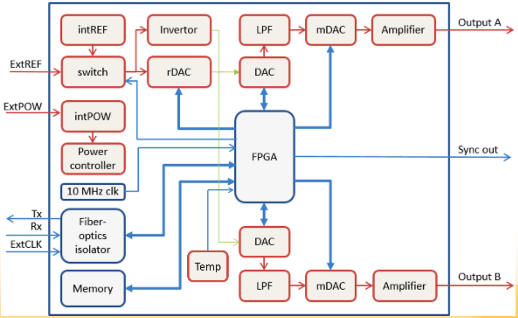
Allan deviation of 1:1 ratio measurement of QHR device P743-2-4 at 4.2 K and working point B = 10.15 T against OF12k9 CTU at a frequency of 1 kHz and current of 23 μA

- **Reference standard**:
  - Octofilar resistor (CTU)
  - Transfer Vishay

  [J. Boháček and B. M. Wood, *Metrologia*, vol. 38, no. 3, 2001]

Generator SWG

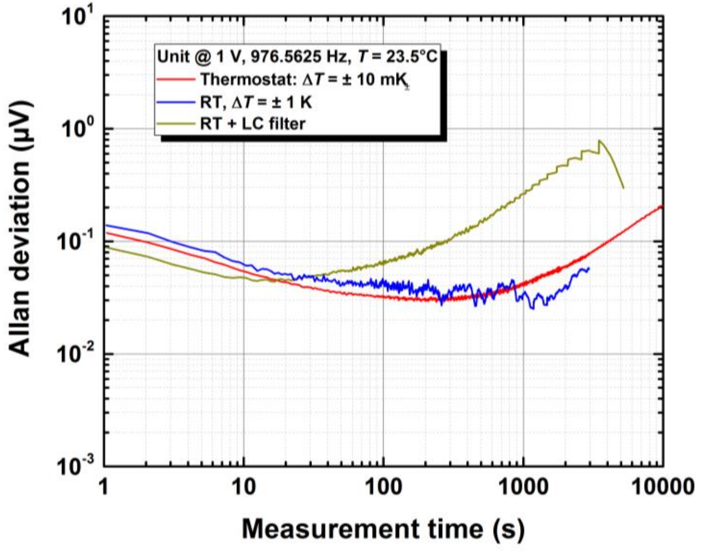
Characteristic	Value
Full scale output voltage (FS)	7 V_{rms} (20 $V_{p\text{-}p}$)
Amplitude resolution	< 0.01 µV/V of FS
Phase resolution	$2{\times}10^{-7}$ rad
Rel. voltage ratio stability of chan. A/B	better than 0.01 $\times 10^{-6}$/30 min.
Frequency range	1 mHz to 20 kHz (100 kHz)
SFDR	> 95 dB @ 100 Hz, > 85 dB @ 1 kHz
Crosstalk between channels A and B	< –150 dB @ 1 kHz
Crosstalk between different modules	Not measurable
Reference clock	10 or 20 MHz Int./Ext.
Reference voltage	10 V_{dc} Int./ 5 to 10 V_{dc} Ext.



[J. Kováč, “Precision low-frequency multichannel generator,” M.S. thesis, Dept. Meas., FEE, CTU Prague, Prague, Czech Republic, 2014.]

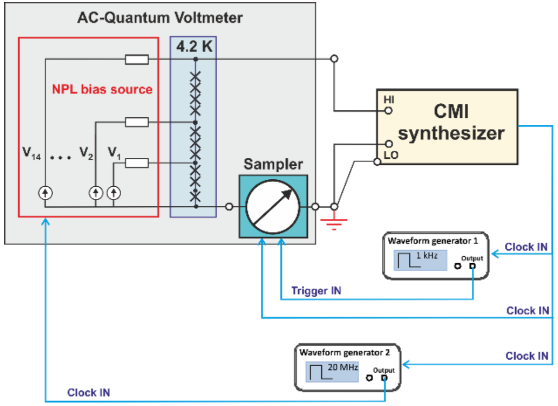
Generator SWG

► Stability of the output voltage



Allan deviation analyses for f = 976 Hz and Vrms = 1 V

PTB's AC quantum voltmeter:

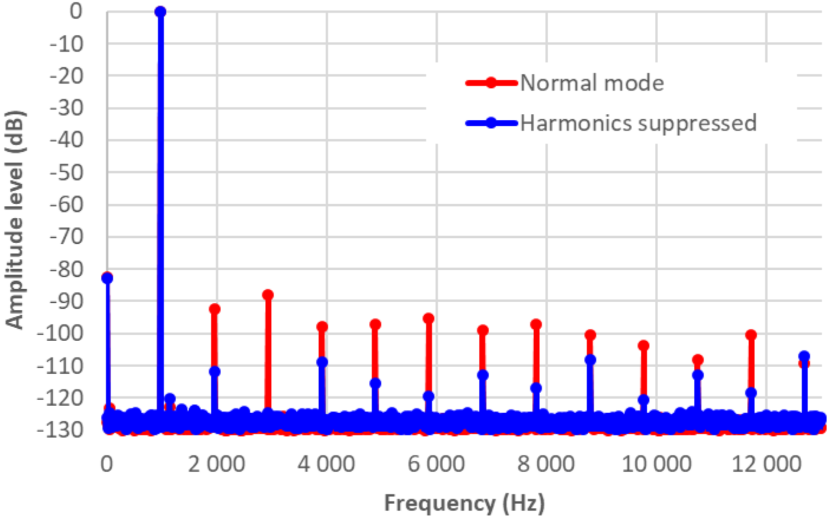


[Kučera at al., 2019, *sent to* IOP MST]

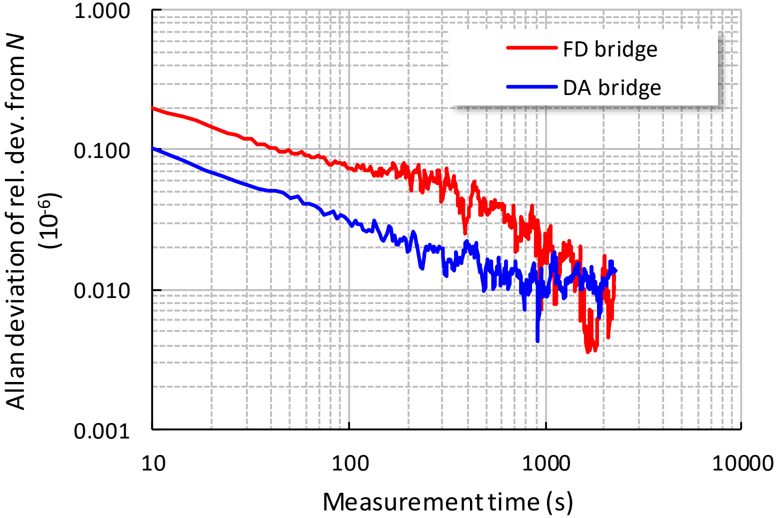
- Long term stability of the output voltage
  - 3 SWG modules with internal voltage reference: ±6 µV/V over 4 years

Generator SWG

► SFDR and bridge stability



An example of the generator spectrum with and without reduction of the first eleven higher harmonic tones for a 1 Vrms signal.



Allan deviation of an N=10:1 ratio measurement of100 pF and 10 pF capacitors at 976 Hz.
The bridge voltage for the DA:1.1 Vrms, FD: 3.5 Vrms.

Verification of the bridge

► **In-phase and quadratic component with known values R-R, C-C @1 kHz**

- Calculable standards
- Known reference standards

Ratio *N*	**Bridge voltage (V_{rms})**	$N_{ref}/N_{nom} - 1$ $\times 10^6$	$N_{FD}/N_{ref} - 1$ $\times 10^6$	$N_{DA}/N_{ref} - 1$ $\times 10^6$
QFR 1 kΩ/QFR 100 Ω	1.1	~ 250	9±34	0.023±0.05
AH11 100 pF/AH11 10 pF	7.0	~0.3	-4.4±34	
	3.8	~0.3	-2.5±34	

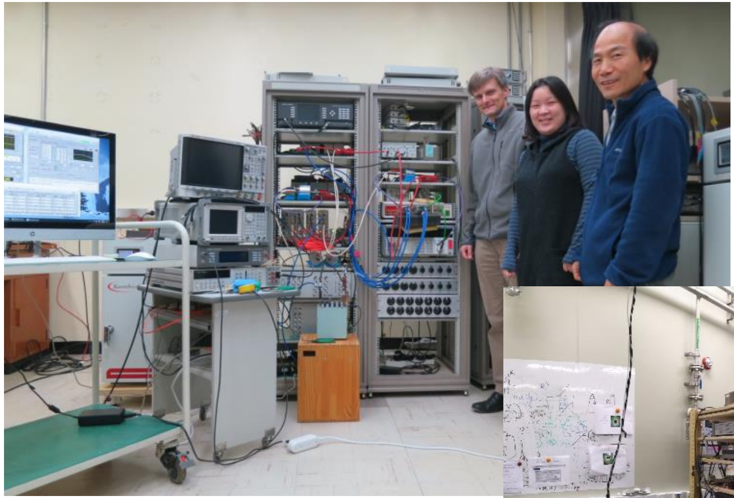
Ratio *N*	τ_{ref} **(ns)**	$\tau_{FD} - \tau_{ref}$ **(ns)**	$\tau_{DA} - \tau_{ref}$ **(ns)**
QFR 1 kΩ/QFR 100 Ω	~ -18	-3.7±4.7	-0.4±1.9

(k=2)

[J. Kučera and J. Kováč, *IEEE TIM* , vol. 67, no. 99,2018]

DigiBridge at NMIs

► **Bridges for KRISS primary Impedance lab (2018)**

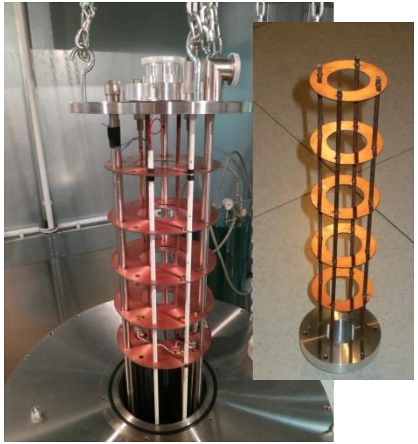


… and KRISS Quantum lab (2020)

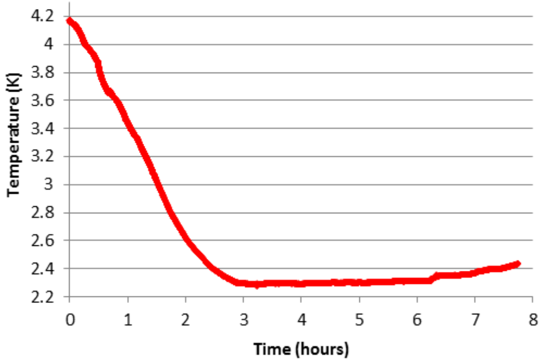
► **Parts at PTB, LNE, VNIIM, NSAI, UME (2019-21)**

Cryostat modification

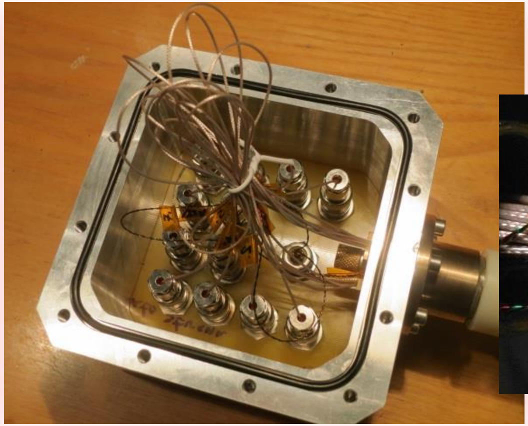
- Original system with 3He insert
- Not suitable for ac measurements

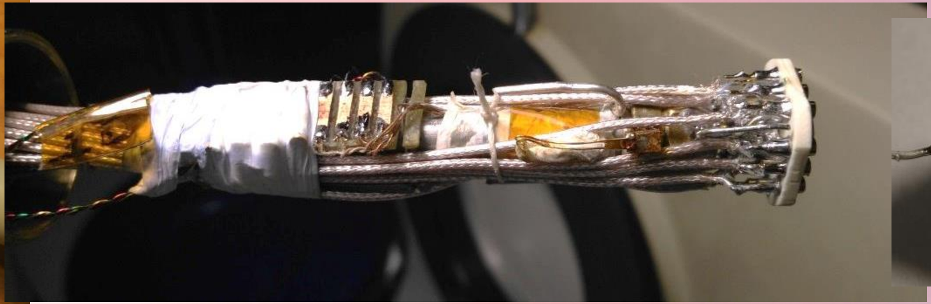


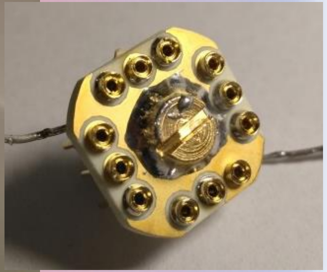
- Removed 3He insert
- Added heat shields
- New probe with coaxial wiring
- Temperature 2.3 – 4.2 K



New cryogenic probe for ac measurements VSM12

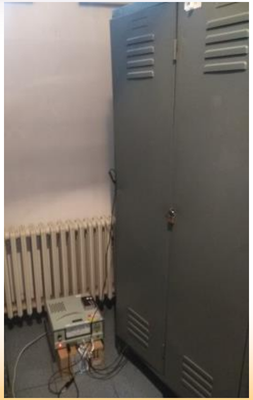




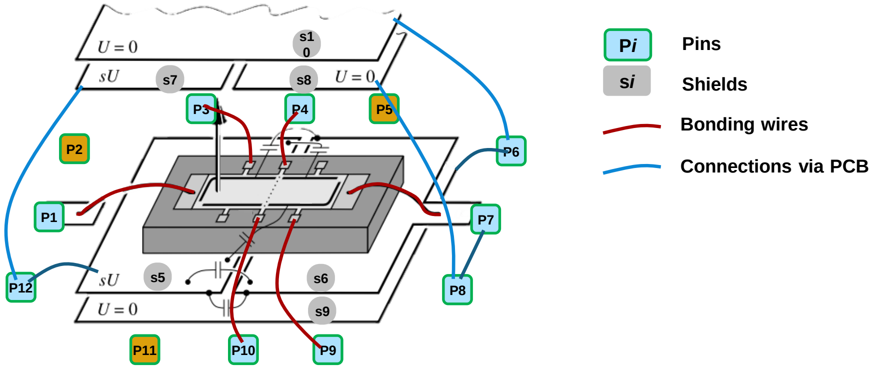


- TO-8 socket compatible with both AC and DC devices
- Isolation between leads > 6×10^{13} Ω
- Isolation between lead and screen > 3×10^{13} Ω
- Parasitic capacitance < 2 aF, D<0.002
- Vacuum sealed

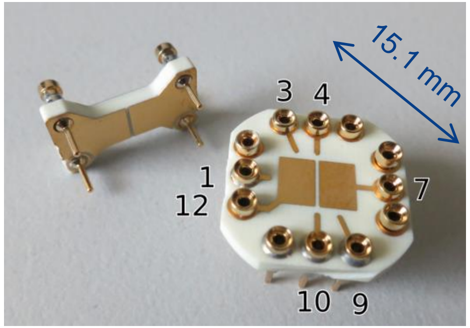
@ 300 K



The ac quantum Hall resistance standard



[B. P. Kibble and J. Schurr, *Metrologia* 45(5) pp. L25–L27, 2008]

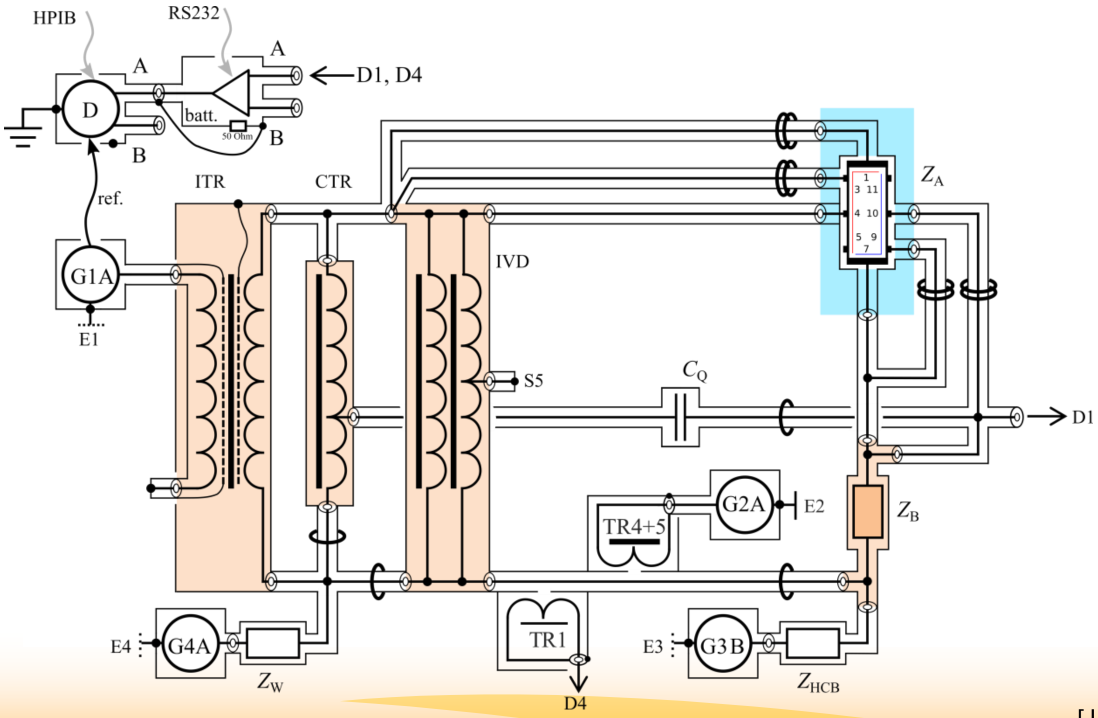


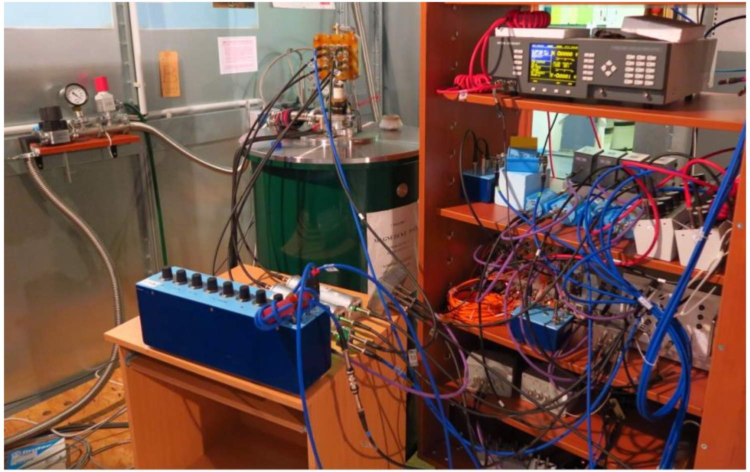
- TO-8 socket compatible chip carrier
- Double shielded technique
- Isolation > 3×10^{13} Ω
- Parasitic capacitance < 30 aF, D<0.003
- GaAs/AlGaAs heterostructure fabricated at PTB
- Working point B ~ 10 T

@ 300 K

DigiBridge for AC QHR

- Modification of 4-TP DA bridge

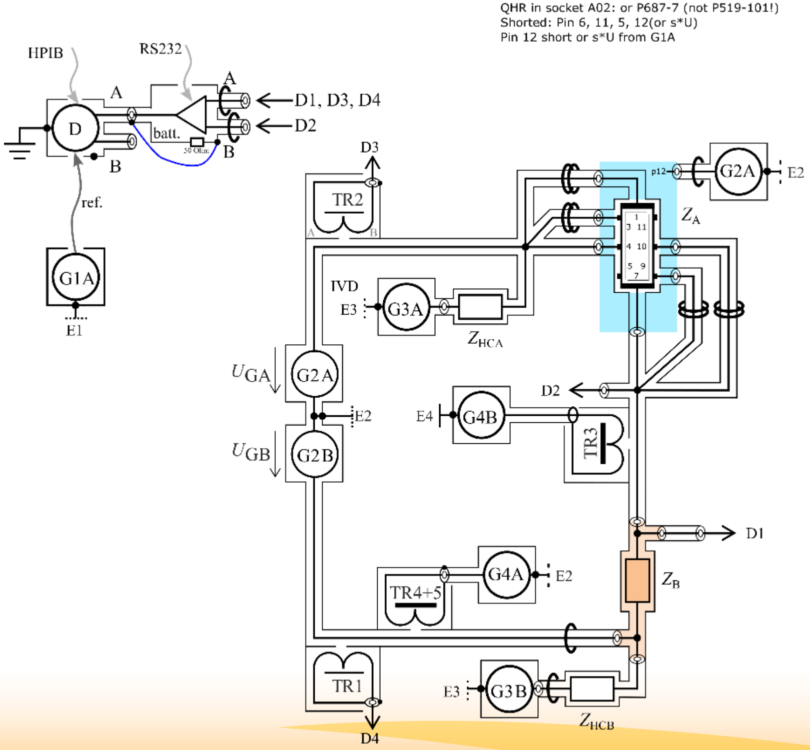


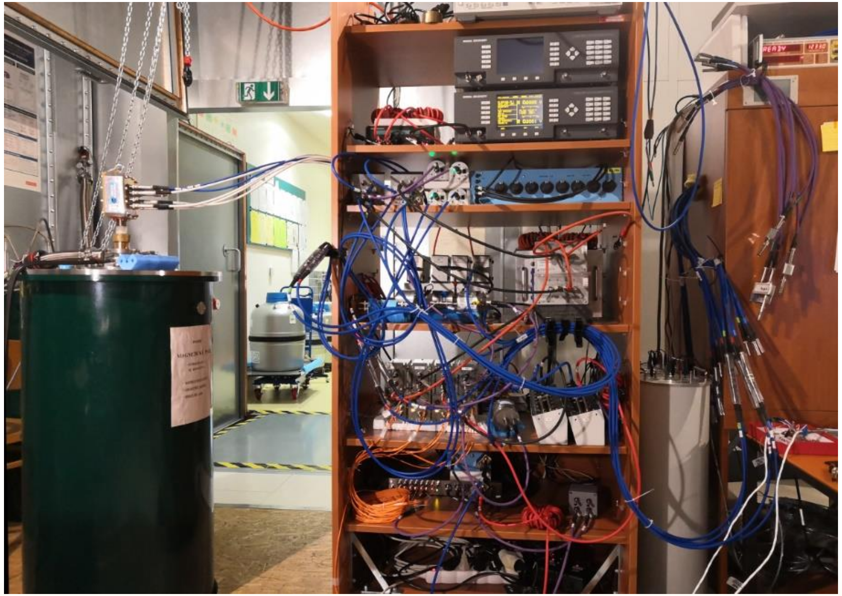


[J. Kučera and J. Kováč, *IEEE Trans. Instr. Meas.*, vol. 67, no. 99, 2018]

DigiBridge for AC QHR

- Modification of 4-TP FD bridge



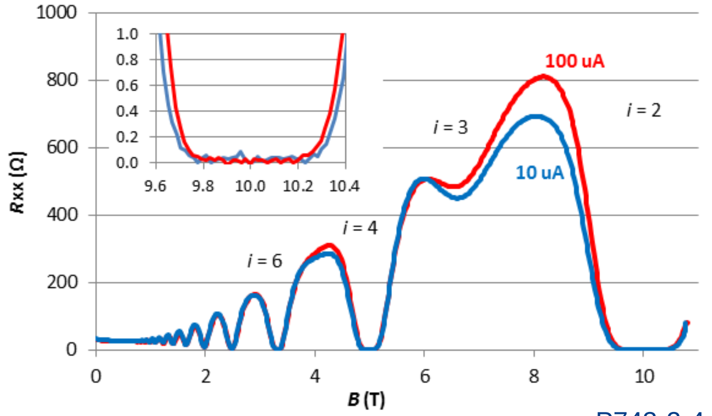


[J. Kučera, P. Svoboda, J. Kováč, K. Pierz, *in preparation*]

Experimental results

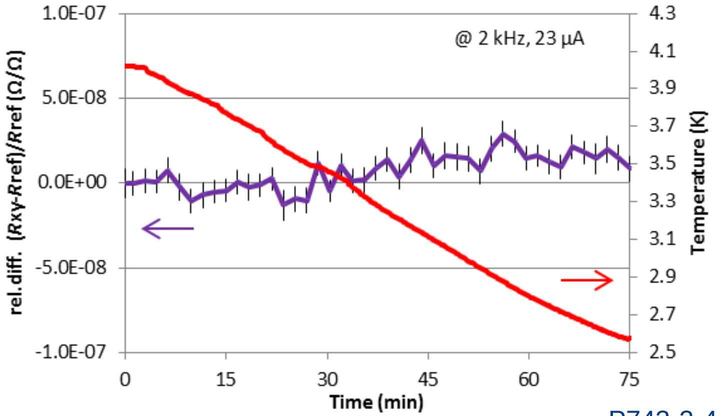
- Quantization check at temperatures 4.2 K … 2.3 K
  - VSM12 probe and QHD P579-101 verified during on-site comparison CMI-BIPM.
    Agreement within (0.6±5) nΩ/Ω (k=2) [P. Gournay *at al. Metrologia* , vol. 54, no. 1A, 2017]
  - Devices P743-2-4 and P743-2-6 compared against well characterized P579-101 (PTB).
    Agreement within 2 nΩ/Ω.

DC: R_{xx} @ 4.2 K



P743-2-4

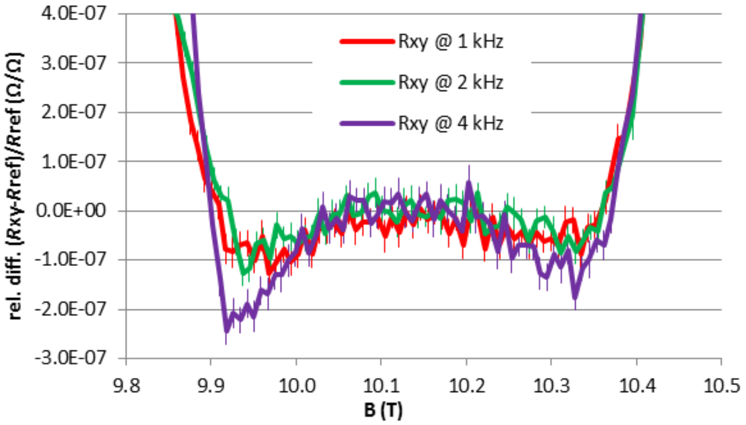
AC: R_{xy} @2 kHz temperature sweep: within ~30 nΩ/Ω



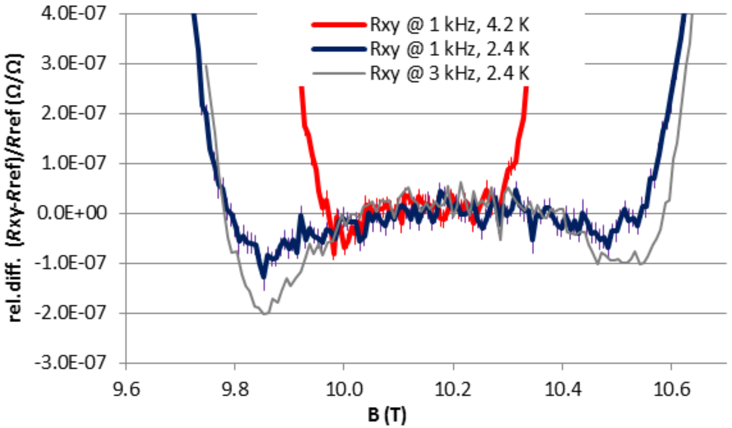
P743-2-4

Experimental results

- Plateau shape at different frequencies: …. and temperatures:



P743-2-4



P743-2-4

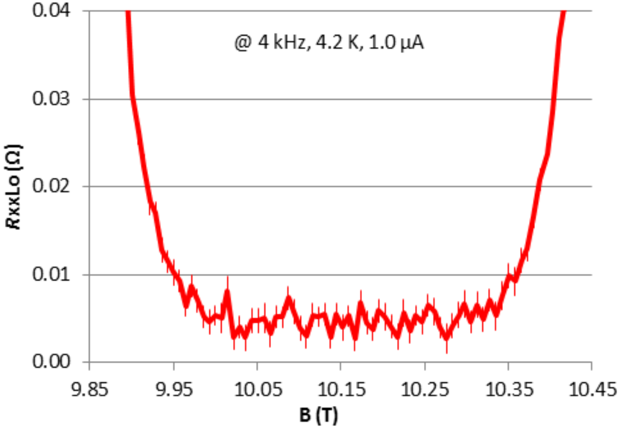
Frequency dependence of R_{xy} plateau shape measured at temperature 4.2 K, current 12 μA (each R_{ref} shifted).

R_{xy} plateau shape at different temperatures and frequencies measured at current 23 μA (each R_{ref} shifted).

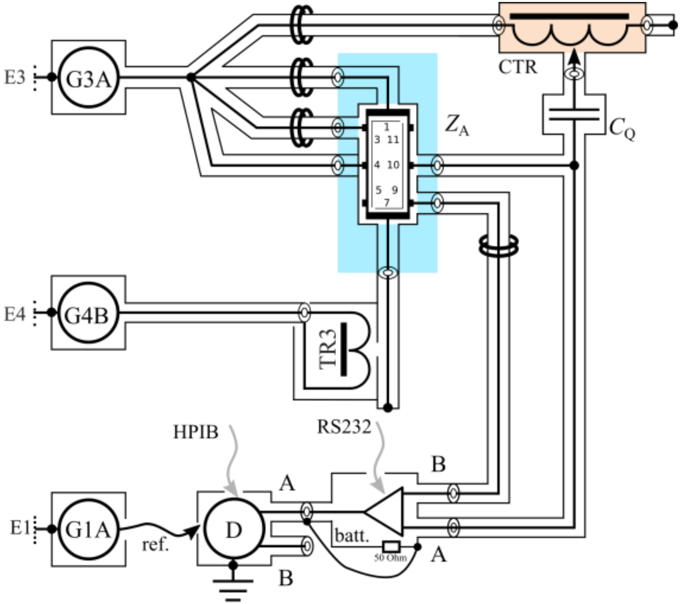
(u_A *with cov. prob. ~95 %*).

Experimental results

- Ac longitudinal resistance R_{xxLo}:



P743-2-4



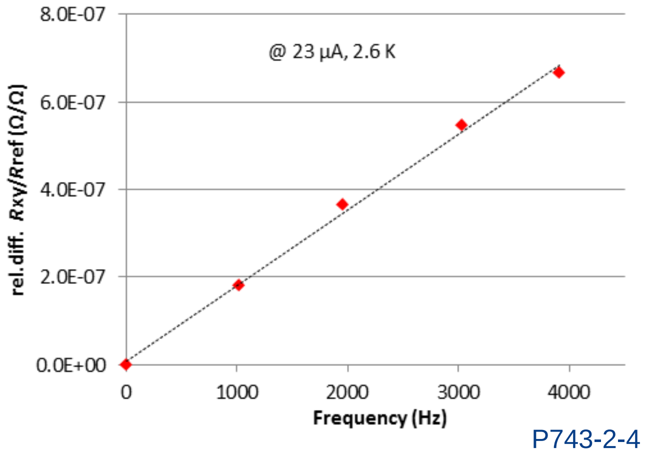
- Ac and dc contact resistance:

$R_{contact}$ < 3 Ω @4.2 K for $I \leq 5$ µA

(u_A with cov. prob. ~95 %).

Experimental results

- Frequency dependence of QHR – without applying shield potential



Frequency dependence of the Hall resistance.

- Slope of about +0.17 µΩ/Ω/kHz

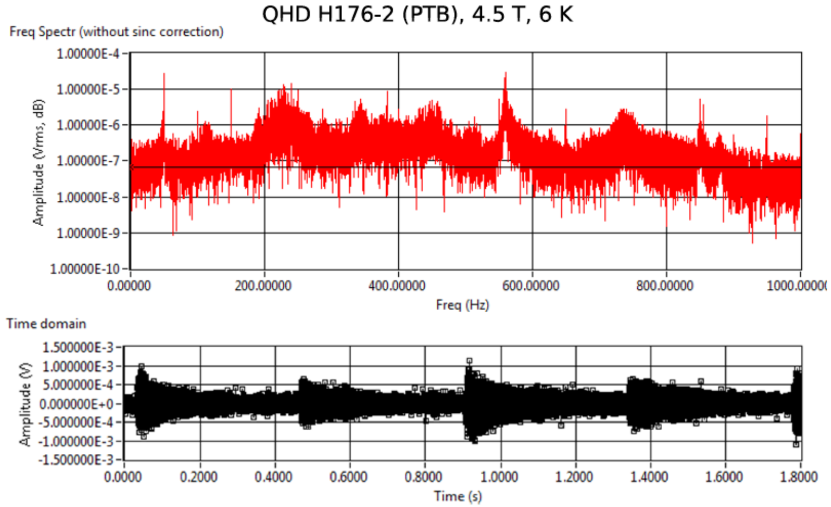
- Resonances in cryogenic part of system immersed in LHe?

Graphene based QHR and cryocooling

GraphOhm

- Modular Lhe-free system
  - Cryocooled magnet with bore at room temperature
  - Cryocooled custom vacuum vessel
  - Magnet field up to 4.5 T, temperature of QHR down to 5 K

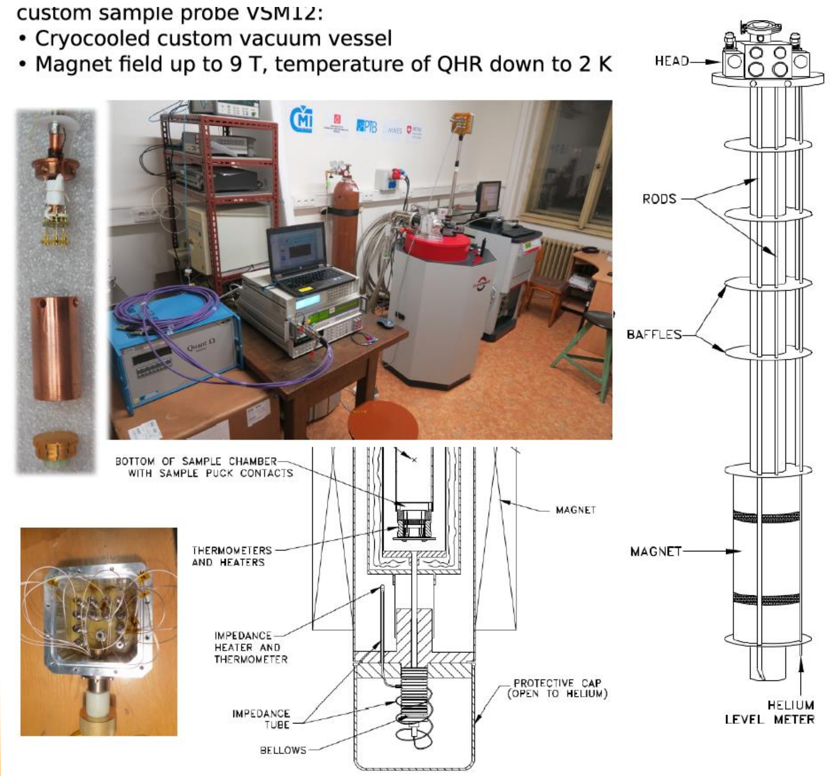
Magnet with cryocooler
Custom vacuum vessel and cryocooler



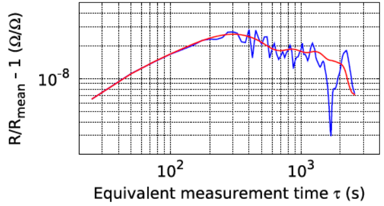
[J. Kučera, M. Šíra, J. Kaštil, P. Fitl, P. Svoboda 2016 Closed cycle refrigerators and their application for realization of QHR, EMRP dissemination meeting, Prague]

Graphene based QHR and cryocooling

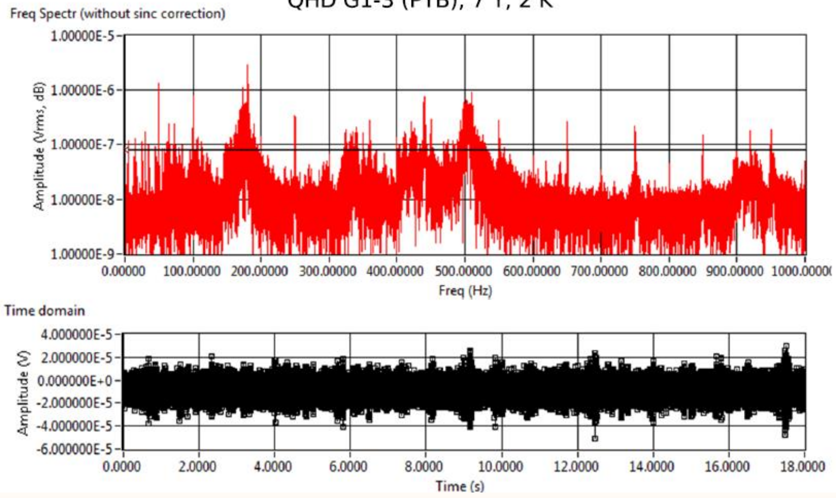
- Closed cycle system Evercool II (Quantum Design, Inc.)



Allan deviation (**blue** line) and overlapped Allan dev. (**red** line) of QHR sampled compared to an artefact. 200 measured points (80 minutes).



QHD G1-3 (PTB), 7 T, 2 K



Summary

- **Digitally assisted/Fully digital DigiBridge**
- **Cryogen-free system investigated**
- **Measurement system for ac QHR measurements developed**
  - Cryogenic part
  - Digitally assisted bridge
- **Robustness of GaAs based devices with appropriate handling, even at temperatures of 2.3 … 4.2 K shown**
  - Dc quantization within few parts in 10^9 @4.2 K, 10 T
  - "Good" ac quantization @2.6 K, 10 T
- **Ongoing work:**

Realization of R-C linkage directly to AC QHR with modified DigiBridge
Graphene based AC QHR under development (EMPIR GIQS)

AC JVS on-site comparison under development with PTB and BIPM

?New EMPIR „Advanced Classical Standards for Electrical Metrology "?

Acknowledgment

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Thank you for your attention

For details:

J. Kučera, J. Kováč, L. Palafox, R. Behr, L. Vojáčková, "Characterization of a precision modular sine wave generator," *sent to IOP MST*

J. Kučera and J. Kováč, "A Reconfigurable Four Terminal-Pair Digitally Assisted and Fully Digital Impedance Ratio Bridge," *IEEE TIM* , vol. 67, no. 99, pp. 1–8, 2018

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Ortolano, M. at al. „An international comparison of phase angle standards between the novel impedance bridges of CMI, INRIM and METAS" *Metrologia*, IOP Publishing, 2018, 55, 499-512

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